EDU Journal of International Affairs and Research (EJIAR), ISSN: 2583-9993
Volume 4, Issue 2, April-June, 2025, Available at: https://edupublications.com/index.php/ejiar

Dynamic Stability of Delaminated Cross ply Composite Plates and Shells

Rambha Thakur[1], Dinesh[2]

[1]Assistant Professor, Department of Civil Engineering, Rattan Institute of Technology and Management, Haryana, India
[2]Research Scholar, Department of Civil Engineering, Rattan Institute of Technology and Management, Haryana, India

ABSTRACT

Fibre reinforced composite plates and shells are increasingly replacing traditional metallic ones. The manufacturing process and service of the composite laminates frequently lead to delamination. Delamination reduces the stiffness and strength of composite laminates because they allow out of plane displacement of plies to occur more easily. Dynamic stability analysis is an integral part of most engineering structures. The present work deals with the study of the effects of free vibration, buckling and dynamic stability of delaminated cross ply composite plates and shells. A first order shear deformation theory based on finite element model is developed for studying the instability region of mid plane delaminated composite plate and shell. The basic understanding of the influence of the delamination on the natural frequencies, non- dimensional buckling load and non-dimensional excitation frequency of composite plates and shells is presented. In addition, other factors affecting the vibration, buckling and dynamic instability region of delaminated composite plates and shells are discussed. The numerical results for the free vibration, buckling and dynamic stability of laminated cross-ply plates and shells with delamination are presented. As expected, the natural frequencies and the critical buckling load of the plates and shells decrease with increase in delamination. Increase in delamination also causes dynamic instability regions to be shifted to lower excitation frequencies.

INTRODUCTION

Composite laminates are widely used in engineering structures due to their excellent properties, such as high strength-to-weight ratio, high stiffness-to-weight ratio and design versatility etc. Delamination can cause serious structural degradation. Which is a de bonding or separation between individual plies of the laminate, frequently occurs in composite laminates. Delamination may arise during manufacturing (e.g., incomplete wetting, air entrapment) or during service (e.g., low velocity impact, bird strikes). They may not be visible or barely visible on the surface, since they are embedded within the composite structures. However, the presence of delamination may significantly reduce the stiffness and strength of the structures and may affect some design parameters such as the vibration characteristic of structure of structure. (e.g., natural frequency and mode shape). Delaminations reduce the natural frequency, as a direct result of the reduction of stiffness, which may cause resonance if the reduced frequency is close to the working frequency. It is therefore important to understand the performance of delaminated composites in a dynamic environment. The subject of predicting the dynamic and mechanical behaviour of delaminated structures has thus attracted considerable attention.

Plate and Shell members have often been used in modern structural systems, because the desired performance can be achieved by controlling the shape of those structures. In particular, high-performance applications of laminated composites to plate and shell members are advantageous because of their light weight, high specific stiffness and high specific strength. However, laminated plates and shells subjected to in plane periodic forces may lead to dynamic instability for certain combinations of load amplitude and disturbing frequency. Furthermore, plates and shell members with de lamination may result in significant changes to their dynamic characteristics. Therefore, it is essential to study the effect of de laminations simultaneously on the dynamic stability of layered shells under periodic loads. Composite plates and shells are widely used in aerospace structures. These are often subjected to defects and damage from both in-service and manufacturing events. Delamination is the most important of these defects.

Importance Of The Stability Studies Of Delaminated Composite Shell

Structural elements subjected to in-plane periodic forces may lead to parametric resonance, due to certain combinations of the values of load parameters. The instability may occur below the critical load of the structure under compressive loads over wide ranges of excitation frequencies. Several means of combating resonance such as damping and vibration isolation may be inadequate and sometimes dangerous with reverse results. Structural elements subjected to in-plane periodic forces may induce transverse vibrations, which may beresonant for certain combinations of natural frequency of transverse vibration, the frequency of the in-plane forcing function and magnitude of the in-plane load. The spectrum of values of parameters causing unstable motion is referred to as the regions of dynamic instability or parametric resonance. Thus the parametric resonance characteristics are of great importance for understanding the dynamic systems under periodic loads.

Delamination between plies is one of the most common defects encountered in composite laminates. Delamination can cause serious structural degradation. Which is a debonding or separation between individual plies of the laminate, frequently occurs in composite laminates. Delamination may arise during manufacturing (e.g.,in complete wetting,air entrapment)or

during service (e.g., low velocity impact, bird strikes). They may not be visible or barely visible on the surface, since they are embedded within the composite structures .However, the presence of delaminations may significantly reduce the stiffness and strength of the structures and may affect some design parameters such as the vibration characteristic of structure of structure. (e.g., natural frequency and mode shape). Delaminations reduce the natural frequency, as a direct result of the reduction of stiffness, which may cause resonance if the reduced frequency is close to the working frequency.

LITERATURE REVIEW

Introduction:-Thus the dynamic stability characteristics are of great technical importance for understanding the dynamic systems under periodic loads. In structural mechanics, dynamic stability has received considerable attention over the years and encompasses many classes of problems. The distinction between “good” and “bad” vibration regimes of a structure subjected to in-plane periodic loading can be distinguished through a simple analysis of dynamic instability region. In modelling delamination, both, analytical as well as numerical methods have been used in studying the dynamic and buckling behaviour of composite laminates. Bolotin (1964) in his text on dynamic stability gives a thorough review of the problems involving parametric excitation of structural elements. The dynamic instability of composite plates and shells without delamination has been studied previously by a host of investigators. The studies in this chapter are grouped into three major parts as follows:

- Vibration
- Buckling
- Dynamic stability

The available literature on dynamic stability of delamination composite plates and shells is very limited.

Free Vibration:
Tenek (1993) *et al.* studied vibration of delaminated composite plates and some applications to non-destructive testing; he studied the impact of delamination on the natural frequencies of composite plates, as well as delamination dynamics over a broad frequency range, using the finite element method based on the three-dimensional theory of linear elasticity. For the case of cantilever laminated plates, the method is successfully compared with experimental observations.

Lee and Lee (1995) examined the free vibration of composite plates with delamination around cut-outs. They presented a finite element approach is to analyze the free vibration of square and circular composite plates with delamination around internal cut-outs. They presented numerical examples including composite plates with delaminations around circular holes or square cut-outs. And they discussed the effects of the cut-outs and the delamination around the cut-outs on the natural frequencies and mode shapes.

Ju *et al* (1995) presented finite element analysis of free vibration of delaminated composite plates. His study was based on Mindlin plate theory. He presented a finite element formulation for the analysis of the free vibration of composite plates with multiple delamination.

Chang (1998) investigated the vibration analysis of a delaminated composite plate subjected to the axial load. The concept of continuous analysis was used to model the delaminated composite plate as a plate on a discontinuous elastic foundation. The elastic adhesive layer between the buckled composite plate and the unreformed substructure working as a foundation to the plate is represented by linear parallel springs.

Williams and Addessio (1998) studied a dynamic model for laminated plates with delimitations. They presented a dynamic, higher-order theory for laminated plates based on a discrete layer analysis. The formulation includes the effects of delaminations between the layers of the plate. The model implements a generalized displacement formulation at the lamina level. The governing equations for the lamina are derived using vibrational principles. Geubelle and Baylor (1998) observed Impact-induced delamination of composite susinga 2D simulation. The delamination process in thin composite plates subjected to low-velocity impact is simulated using a specially developed 2D cohesive/volumetric finite element scheme. Cohesive elements are introduced along the boundaries of the inner layers and inside the transverse plies to simulate the spontaneous initiation and propagation of transverse matrix cracks and delamination fronts.

Hou and Jeronimidis (1999) presented vibration of delaminated thin composite plates. He studied experimental (employed free-free vibration system) and finite element modelling of vibration of GFRP laminated circular plates. He shows that the resonant frequencies of low velocity impacted plates are functions of matrix cracking and the local thickening coupling with inter laminar delamination.

Ostachowicz and Kaczmarezyk (2001) studied the vibration of composite plates with SMA fibres in a gas stream with defects of the type of delamination. They analyzed the dynamics of a multi-layer composite plate with delaminations subjected to an aerodynamic load. They proposed finite element model to predict the dynamic response of the system with embedded shape memory alloys (SMA) fibres. Hu *et al* (2002) investigated vibration analysis of delaminated composite beams and plates using a higher-order finite element. In order to analyze the vibration response of delaminated composite plates of moderate thickness, heproposeda FEM Model based on a simple higher- order plate theory, which can satisfy the zero transverse shear strain condition on the top and bottom surfaces of plates.

Chen *et al* (2004) examined the dynamic behaviour of delaminated plates considering progressive failure process. A formula of element stiffness and mass matrices for the composite laminates is deduced by using the first-order shear deformation theory combined with the selecting numerical integration scheme.

Lee and Chung (2010) observed the finite element delamination model for vibrating composite spherical shell panels with central cut-outs. They developed finite element model of vibrating laminated spherical shell panels with delamination around a central cut-out is based on the third-order shear deformation theory of Sanders. In the finite element formulation for the delamination around cut-out, the seven degrees of freedom per each node are used with transformations in order to fit the displacement continuity conditions at the delamination region.

Static Stability:

Hwang and Mao (2001) investigated failure of delaminated carbon/epoxy composite plates under compression. Their work is to study the buckling loads, buckling modes, post buckling behaviour, and critical loads of delamination growth for delaminated unidirectional carbon/epoxy composites. Non linear buckling analysis, which is a finite-element method including contact elements to prevent the overlapping situation, was applied to predict the delamination buckling loads.

Parhi *et al*. (2001) presented hygro thermal effects on the dynamic behaviour of multiple delaminated composite plates and shells. They presented a quadratic iso parametric finite element formulation based on the first order shear deformation theory for the free vibration and transient response analysis of multiple delaminated doubly curved composite shells subjected to a hygrothermal environment. For the transient analysis Newmark's direct integration scheme is used to solve the dynamic equation of equilibrium at every time step.

Liu and Yu (2002) studied the Finite element modelling of delamination by layer wise shell element allowing for interlaminar displacements. In his paper, he gave a brief review of the layer wise shell element.

Wang *et al.* (2003) observed Non-Linear Thermal Buckling for Local Delamination near the Surface of Laminated Plates. An investigation is carried out to understand the non-linear thermal buckling behaviour of local delamination near the surface of fibre reinforced laminated plates. The shape of the delaminated region considered is rectangular, elliptic or triangular..

Tafreshi (2006) examined Delamination buckling and post buckling in composite cylindrical shells under combined axial compression and external pressure. He investigated the characteristics of the buckling and post buckling behaviour of delaminated composite cylindrical shells .The combined double-layer and single-layer of shell elements are employed which in comparison with the three dimensional finite elements requires less computing time and space for the same level of accuracy.

Yang and Fu(2006) investigated the Delamination growth of laminated composite cylindrical shells. They derived the post buckling governing equations for the laminated cylindrical shells, based on the variational principle of moving boundary, and the corresponding boundary and matching conditions are given. At the same time, according to the Griffith criterion, the formulas of energy release rate along the delamination front were obtained and the delamination growth was studied.

Oh *et al*. (2008) observed buckling analysis of a composite shell with multiple delaminations based on a higher order zig-zag theory. They developed a new three-node triangular shell element based on a higher order zig-zag theory for laminated composite shells with multiple delaminations.The base line higher order zig-zag composite shell theory for multiple

delaminations has been developed in a general curvilinear coordinate system and in general tensor notation.

Chirica *et al.* (2010) investigated the buckling analysis of the composite plates with delaminations. In this paper they studied of the influence of elliptical delamination on the changes in the buckling behaviour of ship deck plates made of composite materials is treated. A delamination model has been developed by using the surface-to-surface contact option, in licensed FEM code COSMOS/M. So, the damaged part of the structures and the undamaged part were represented by well-known finite elements (layered shell elements).

Dynamic Stability:
Radu and Chattopadhyay (2000)analyzed the dynamic stability of composite plates including delamination using higher order theory and transformation matrix. He analyzed composite plates with various thickness and delamination length and placement. Delamination affects the instability regions by shifting them to lower, parametric resonance frequencies and by modifying.

Jinhua and Yiming (2007) studied the analysis of dynamic stability for composite laminated cylindrical shells with delaminations. they derived a set of dynamic governing equations for the delaminated cylindrical shells By introducing the Heaviside step function into the assumed displacement components and using the Rayleigh–Ritz method for minimizing the Total potential energy ,Then ,the dynamic governing equations are written as the Mathieu-type equations to describe the parametric vibrating behaviour of the shells, and these equations are solved by employing the Bolotin method.

Park and Lee (2009) examined parametric instability of delaminated composite spherical shells subjected to in-plane pulsating forces. The dynamic stability analysis of delaminated spherical shell structures subjected to in-plane pulsating forces is carried out based on the higher-order shell theory of Sanders.

Aim and Scope of present study:
The present study deals with the effect of delamination on dynamic characteristic of composites.

The Various Modules Are:

- Vibration of delaminated composite shells
- Buckling of delaminated composite shells
- Dynamic stability of delaminated plates and shells

MATHEMATICAL FORMULATION

The Basic Problems
This chapter involves the finite element mathematical formulation for vibration, static and dynamic stability analysis of the plate and shell structures of various geometry with/without delamination. The basic configuration of the problem considered here is a doubly curved panel with mid plane single delamination subjected to in plane periodic load. The boundary conditions are incorporated in the most general manner.

1.1.Proposed Analysis
The governing equations for the dynamic stability of the delaminated composite plates and shell are derived on the basis of first order shear deformation theory .The element stiffness, geometric stiffness and mass matrices are derived on the basis of principle of minimum potential energy and Lagrange‟s equation.

Governing equation for analysis:-

Equation for free vibration is, $$([\boldsymbol{K}] - \boldsymbol{\omega}^2 [\boldsymbol{M}])\{\emptyset\} = \{\mathbf{0}\}$$

Where$[K]$ and $[M]$are the global stiffness and global mass matrices,$\omega$is the natural frequency and $\emptyset$is the corresponding eigen vectors i.e. mode shape.
Equation for buckling analysisis**,**

$$([\boldsymbol{K}] - \boldsymbol{P}[\boldsymbol{K}_g])\{\emptyset\} = \{\mathbf{0}\}$$

Where$[K]$and$[K_g]$are the bending stiffness matrix and geometric stiffness matrix. The Eigen values of the above

equation gives the buckling loads for different modes. The lowest value of buckling load (P) is termed as critical buckling load $(P_{cr})$of the structure.
Equation for dynamic stability analysis in line with [Radu and Chattopadhyay(2000)],

Where M,K and

$$K + (\alpha_0 \pm 0.5\alpha_1)P_{cr}K_G - 0.25M\theta^2 = 0$$

K_G are the mass, the stiffness and the geometric matrices.θRepresent the frequency,α_0
And $\alpha_1$are static and dynamic parameters taking values from 0to 1.

RESULTS AND DISCUSSIONS

Introduction:
Recently composite plates and shells as a structural element are widely using in aerospace, civil, mechanical and other engineering structures and playing a important role in such kind of industries.

In this chapter, the result of vibration, buckling and dynamic stability of plate and shell structures with and without delamination is presented by using above FEM formulation. The instability regions are determined for composite plates and shell with and without delamination and the results are presented for different delamination lengths and percentage. The study is aimed upon the following studies.

- Comparison with previous studies
- Numerical Results

Boundary conditions:
Numerical results are presented for delaminated composite plates/shells with different combination of boundary conditions. Shells of various geometry such as spherical (R_y/R_x=1) and cylindrical (R_y/R_x=0) are studied.

Comparison with previous studies:
The vibration, buckling of plates/shells and dynamic stability results of plates based on present formulation are compared with that of existing literature.

Vibration of composite plates and shells
The result on free vibration of cross-ply delaminated plates and shell are compared with result by parhi*etal* (2002)using first order shear deformation theory .

For this free vibration analysis, the boundary conditions used in present study is simply supported and composite (0/90/0/90) spherical and cylindrical shell and plates with delamination is carried out with the following geometric and material properties:

Buckling of composite plates:
The present formulation is validated for buckling of cross-ply composite plates with and without delamination with result by Librescu et al (1989), Sciuva and Carrera (1990), Sahu (2001) and Radu and chattopadhyay (2002).

Table 2.1: Comparison of non-dimensional buckling loads of a square simply supported doubly curved panel with (0/90) lamination.

a/b=1,a/h=10,E_{11}=40E_{22}, G_{12}=G_{13}=0.6E_{22},G_{23}=0.5E_{22},ϑ_{12}=ϑ_{23}=0.25

$$\bar{N}_x = N_x \frac{a^2}{E_2\ h^3}$$

Taken value,a=0.5, b=0.5,t=0.05, E_{11}=160GPa, E_{22}=4GPa,G_{12}=G_{13}=2.4GPa, G_{23}=2GPa.

$$\rho = 1600kg/m^3,$$

a= $E_{11} = 134.4GPa.\ E_{22} = 10.34GPa,\ G_{12} = G_{13} = 4.999GPa,\ G_{23} = 1.999GPa$.

127mm,b=12.7mm,h=1.016mm,stackingsequence=(0/90/0/90/90/0/90/0).

Numerical Results:
Numerical validation of the governing equation of the composite plate and shell for vibration and buckling is performed by solving the corresponding free vibration equation and buckling equation eigen value problems. Finally, the dynamic stability phenomenon is investigated for effect of layers of ply, different degree of orthotropic, different static load factor, different length-thickness ratio and aspect ratio.

Numerical results of vibration are presented for simply supported square plate and shell having following Geometry and material properties :a=b=0.5m,a/h=100,R/a=5and R/a=10,
ϑ_{12}=0.25, $\rho = 1600kg/m^3$,

$$E_{11} = 172.5GPa. \; E_{22} = 6.9GPa, \; G_{12} = G_{13} = 3.45GPa, \; G_{23} = 1.38GPa \, .$$

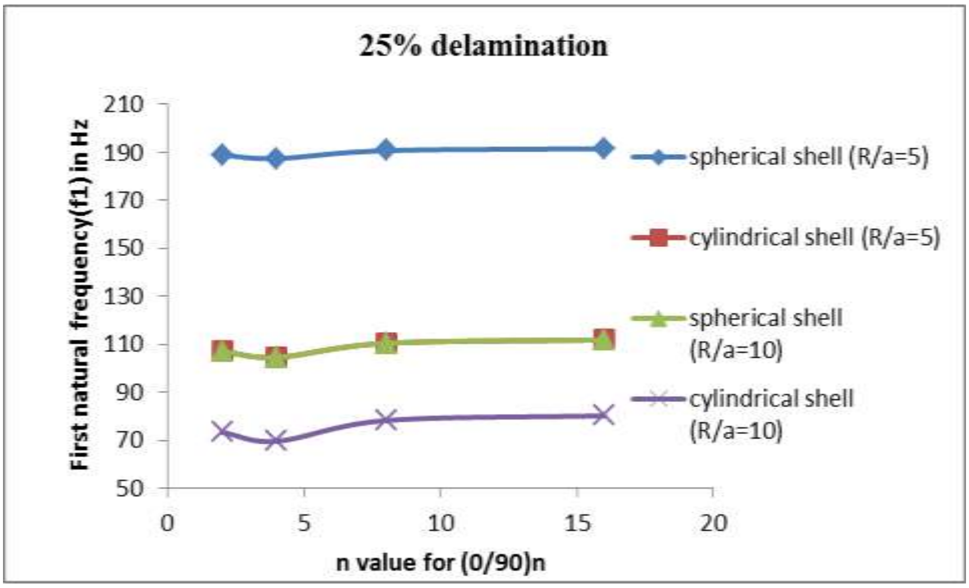

Figure 4: First Natural Frequency Vs. No. Of Layer For Simply Supported Composite Shell With A Single Mid-Plane Delamination.

Natural frequency of composite spherical and cylindrical shell for different aspect ratio at 25% delamination is investigated in Table 3.2.The variation of natural frequency with different aspect ratio is graphically presented in fig 5.

Table 3.2. Natural frequencies (Hz) for 25% delaminated cross ply-(0/90)$_2$ simply supported composite spherical and cylindrical shells with different aspect ratio.

a/b	Spherical shell $R_x/a=5, R_y/b=5$	Cylindricalshell $R_x/a=\infty, R_y/b=5$
0.5	293.8187	220.6784
1	187.4469	104.5625
1.5	153.3563	71.5290
2	134.7433	56.4072

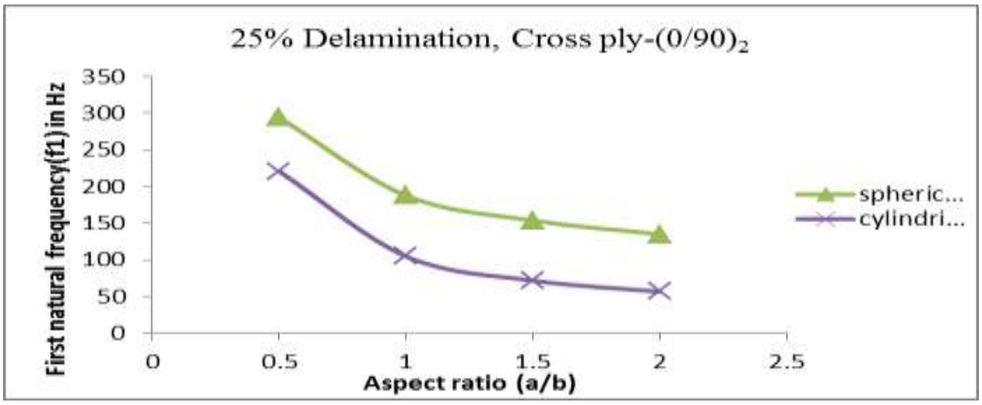

Figure 5.**First Natural Frequency Vs. Aspect Ratio For Simply Supported Composite Shell With A Single Mid-Plane Delamination.**

Table 3.3.**Natural frequencies(Hz) for delaminated crossply-(0/90)$_2$ simply supported composite spherical and cylindrical shells with different b/h ratio for R/a=5.**

%delamination	Sphericalshell			Cylindricalshell		
	b/h=100	b/h=50	b/h=25	b/h=100	b/h=50	b/h=25
0	201.8568	256.4682	400.9718	128.9892	204.7105	371.4012
25	187.4469	208.1226	273.6824	104.5625	138.5553	226.2074
56.25	183.9246	195.9246	237.0790	98.2757	119.4906	179.8898

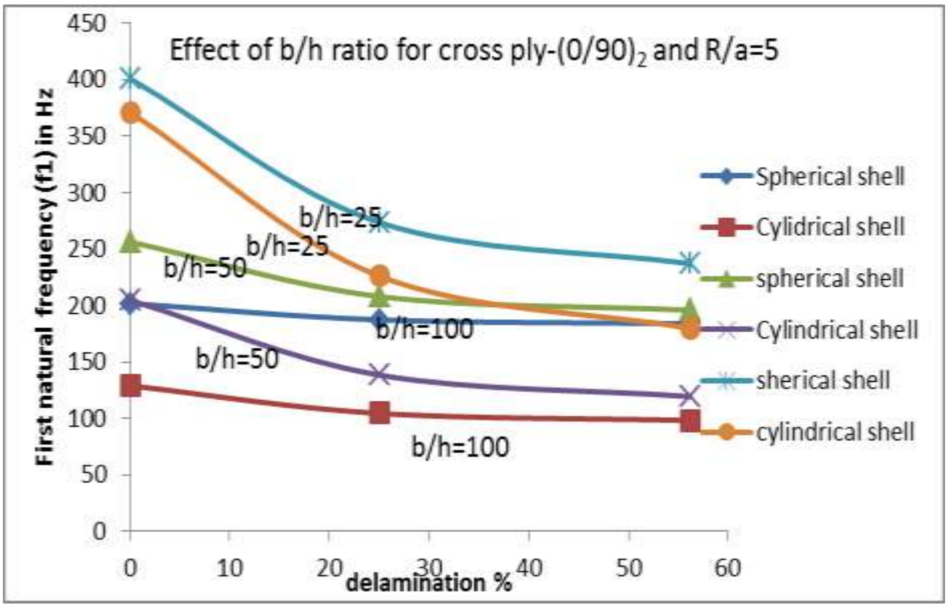

Figure 6.**First natural frequency vs. delamination % at different b/h ratio for simply supported composite shell with a single mid-plane delamination.**

Table3.4 and fig 7 shows that the numerical result of natural frequency of spherical and cylindrical shell with different degree of orthotropic with different % of delamination. Fig 7 represent the natural frequency decreases with % delamination increases at each different degree of orthotropy.

Table 3.4.**Natural frequencies (Hz)for delaminated crossply-(0/90)$_2$simply supported composite spherical and cylindrical shells with different orthotropic ratio for R/a=5.**

E_1/E_2	Sphericalshell %delamination			Cylindricalshell %delamination		
	0	25	56.25	0	25	56.25
10	308.9156	238.2747	216.4470	271.7676	186.6011	157.7208
25	201.8568	187.4469	183.9246	128.9892	104.5625	98.2757
40	470.1998	300.8696	252.0983	445.2868	257.5297	197.9235

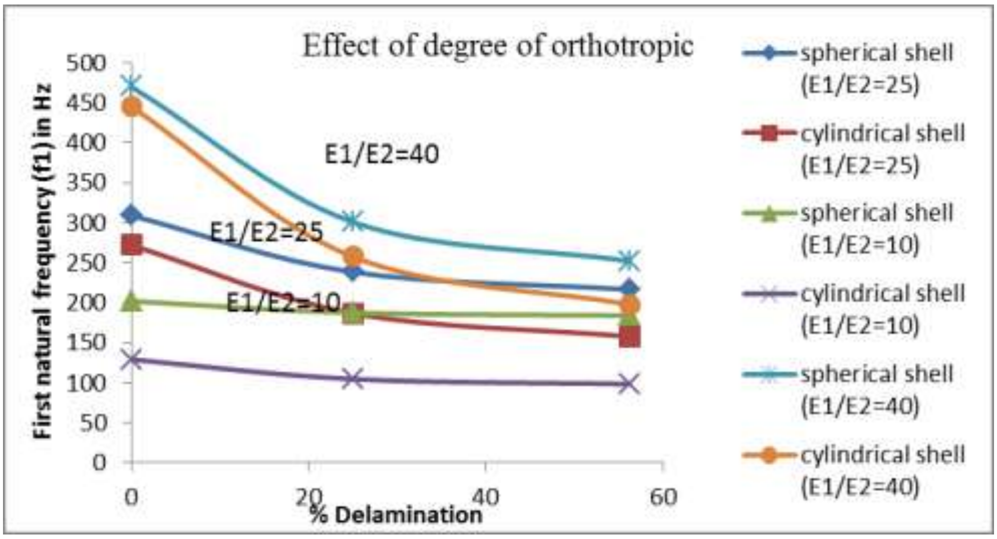

Figure 7. **First natural frequency vs .delamination % at different E1/E2 ratio for simply supported composite shell with a single mid-plane delamination.**

Numerical results of buckling are presented for simply supported square plate and shell. The program of the finite element formulation developed in MATLAB 7.8.0 and validated by comparing the author‟s results with those available in the existing literature. For this stability analysis, the boundary conditions used in present study is simply supported. And composite $(0/90)_n$ spherical and cylindrical shell and plates with and without delamination is carried out with the following geometric and material properties: a/b=1, a/h=10, $E_{11}=40E_{22}$, $G_{12}=G_{13}=0.6E_{22}$, $G_{23}=0.5E_{22}$, $\vartheta_{12}=\vartheta_{23}=0.25$.

Table 4.1: Variation of non-dimensional buckling load with different no. of layers for 0% delaminated composite shell. **$R_x/a=5$, $R_y/a=5$cross-ply-$(0/90)_n$**

No.of layers	Spherical shell	Cylindrical shell	Plate
3	22.0426	21.3819	21.2575
5	24.6990	23.9196	23.8427
7	25.7988	24.8905	24.8387
9	26.5486	25.4525	25.4205

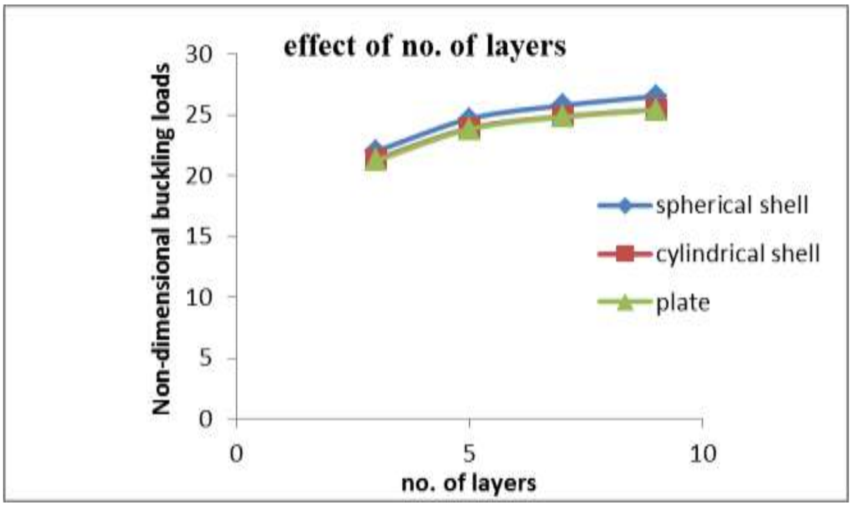

Figure 8: Variation of non-dimensional buckling load vs. no. of layers for simply supported composite shell.R_x/a=5, R_y/a=5, cross-ply-$(0/90)_n$

Table 4.2: Variation of non-dimensional buckling load with different no.of layers for 0% delaminated composite shell.R_x/a=10, R_y/a=10 cross-ply-$(0/90)_n$

No.of layers	Spherical shell	Cylindrical shell	Plate
3	21.4528	21.2887	21.2575
5	24.0540	23.8620	23.8427
7	25.0715	24.8516	24.8387
9	25.6833	25.4284	25.4205

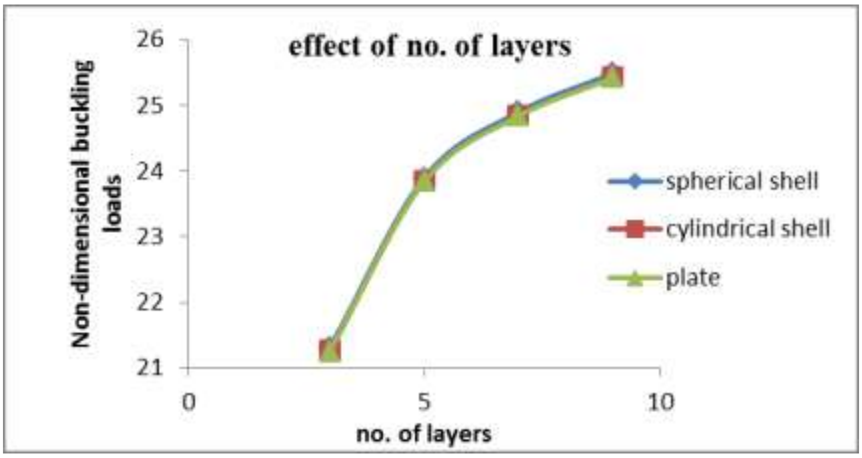

Figure 9: Variation of non-dimensional buckling load vs. no. of layers for simply supported composite shell. R_x/a=10 R_y/a=10, cross-ply-$(0/90)_n$

Table 4.3: Variation of non-dimensional buckling load with different no. of layers for 0% delaminated composite shell. R_x/a=20, R_y/a=20 cross-ply-$(0/90)_n$

No.of layers	Spherical shell	Cylindrical shell	Plate
3	21.3063	21.2633	21.2575
5	23.8954	23.8475	23.8427
7	24.8965	24.8419	24.8387
9	25.4854	25.4225	25.4205

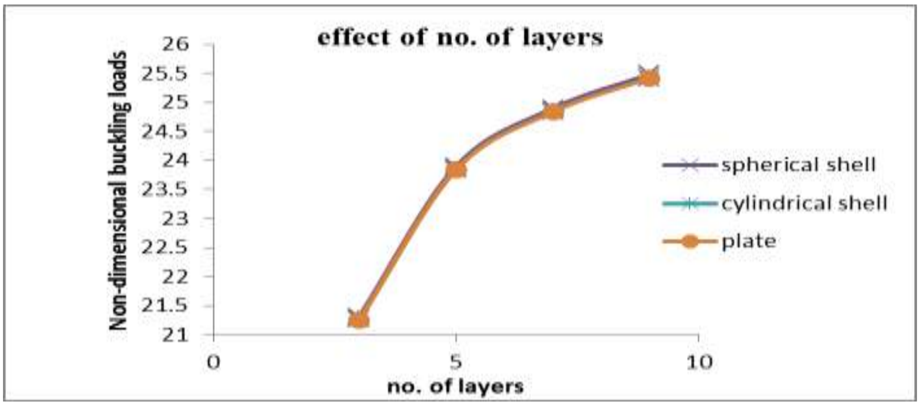

Figure 10: Variation of non-dimensional buckling load vs. no. of layers for simply supported composite shell.R_x/a=20, R_y/a=20,cross-ply-(0/90)$_n$.

Table 4.4 and Table 4.5 represent the numerical result of non-dimensional buckling load with different b/h ratio for 0% delamination of composite shell. The variation of non-dimensional buckling load with different b/h ratio is presented in fig 11 and fig 12. It investigated that as b/h ratio increases the non-dimensional buckling load increases.

Table 4.4: Variation of non-dimensional buckling load with different b/h ratio for 0% delaminated composite shell. R_x/a=5, R_y/a=5, E_1=10E_2,cross-ply-(0/90/0/90/0)

b/h	Sphericalshell	Cylindricalshell
10	6.8671	6.6333
25	10.0591	9.0030
50	14.8080	10.5171

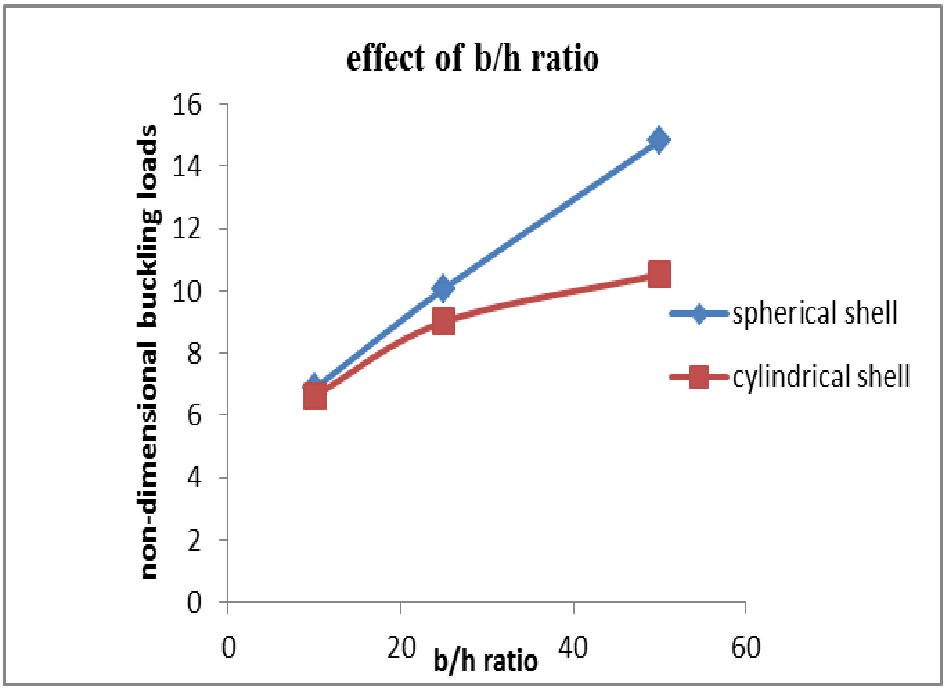

Figure 11:Variation of non-dimensional buckling load vs.b/h ratio for simply supported composite shell.R_x/a=5, R_y/a=5, cross-ply-(0/90)$_n$.

Table 4.5:Variation of non-dimensional buckling load with different b/h ratio for0%delaminated composite shell.R_x/a=10, R_y/a=10, E_1=10E_2, cross-ply-(0/90/0/90/0)

b/h	Spherical shell	Cylindrical shell
10	6.6183	6.6249
25	9.0448	8.7832
50	10.5710	9.5276

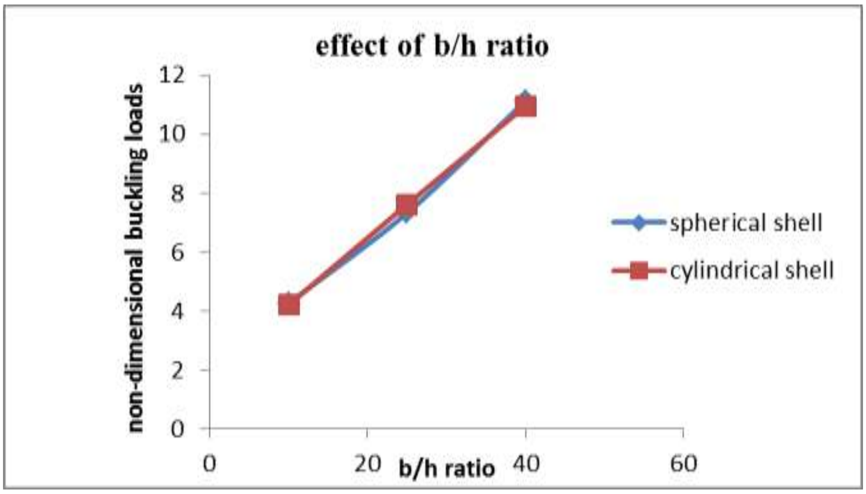

Figure 12: Variation of non-dimensional buckling load vs. b/h ratio for simply supported composite shell.R_x/a=5, R_y/a=5,cross-ply-$(0/90)_n$.

Numerical result of non-dimensional buckling load with different degree of orthotropic for 0% delamination composite shell is presented in table 4.6 and table 4.7. Variations of non- dimensional buckling load with different degree of orthotropy shown in fig13 and fig14.

Table 4.6: Variation of non-dimensional buckling load with different degree of orthotropic for 0% delaminated composite shell. R_x/a=5, R_y/a=5, a/h=10,cross-ply-(0/90/0/90/0)

E_1/E_2	Sphericalshell	Cylindricalshell
10	4.5802	4.2659
25	8.2576	7.7389
40	11.9450	11.1458

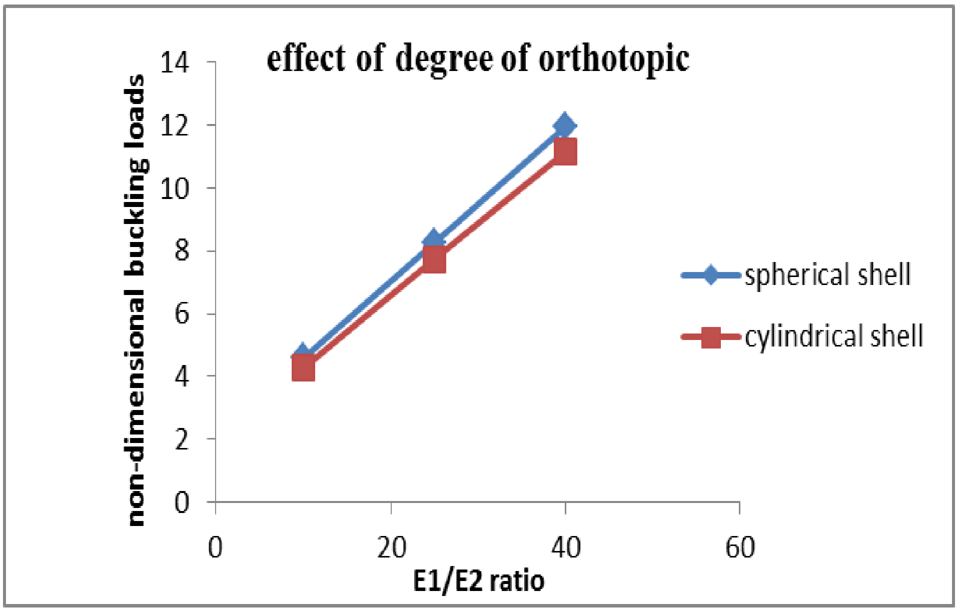

Figure 13: Variation of non-dimensional buckling load vs. E_1/E_2 ratio for simply supported composite shell.$R_x/a=5$, $R_y/a=5$

Table 4.7:Variation of non-dimensional buckling load with different E_1/E_2 ratio for 0% delaminated composite shell.$R_x/a=10$, $R_y/a=10$, a/h=10, cross-ply-(0/90/0/90/0).

E_1/E_2	Spherical shell	Cylindrical shell
10	4.2730	4.2149
25	7.3335	7.6197
40	11.1338	10.9569

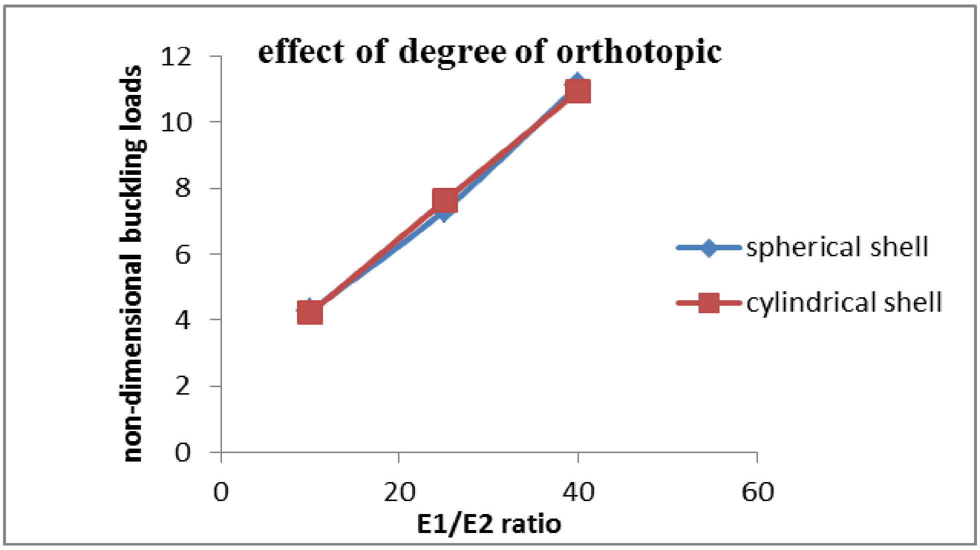

Figure 14: Variation of non-dimensional buckling load vs. E_1/E_2 ratio for simply supported composite shell.$R_x/a=5$, $R_y/a=5$

Numerical results of non-dimensional buckling load with different no of layers for 6.25%, 25% and 56.25% delaminated composite plate and shell with different curvature are presented in Table 4.8, Table 4.9, and Table 5.1. The variation of non-dimensional buckling load with different no. of layers with different% of delamination ispresentedinfig15,fig16, and fig 17

Table 4.8: Variation of non-dimensional critical buckling load with different no. of layers for different percentage of delaminated composite shell.R_y/a=10.Cross-ply-$(0/90)_n$

Noof layers	Delamination%		
	6.25%	25%	56.25%
2	11.0223	10.1426	9.3741
4	14.7349	7.8369	4.7022
8	18.5797	11.7883	8.5228

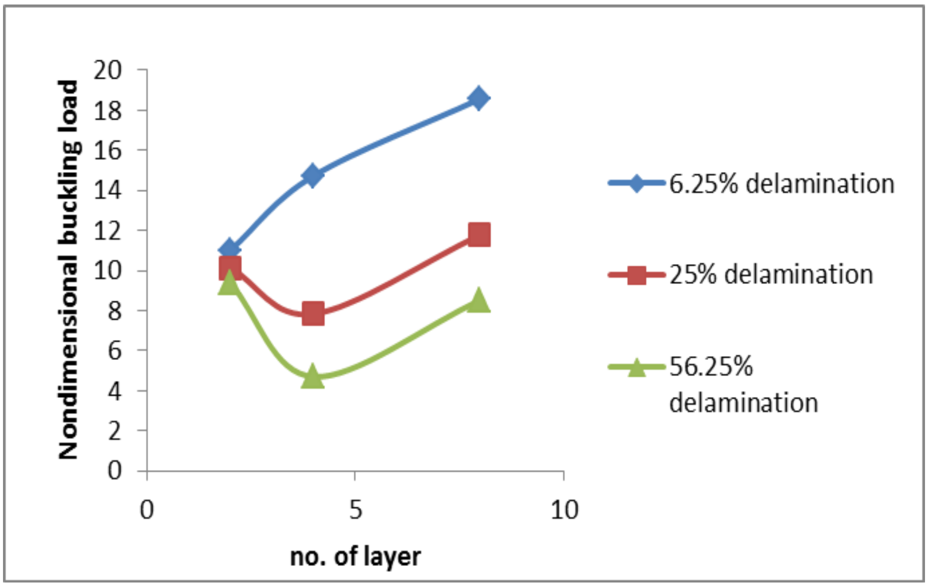

Figure 15: Variation of non-dimensional buckling load vs. no. of layers for simply supported composite shell with different % of delamination. R_y =2,cross-ply-$(0/90)_n$

Table 4.9: Variation of non-dimensional critical buckling load with different no. of layers for different percentage of delaminated composite shell. R_x/a=5, R_y/a=5cross-ply-$(0/90)_n$

Noof layers	Delamination%		
	6.25%	25%	56.25%
2	11.5480	10.6895	9.3741
4	15.2722	8.3798	4.7022
8	9.9338	5.2532	9.0993

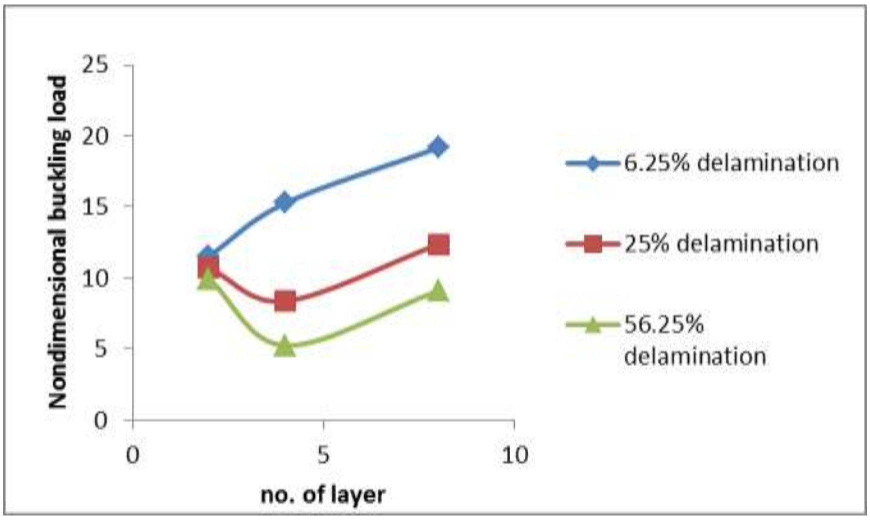

Figure 16: Variation of non-dimensional buckling load vs. no. of layers for simply supported composite shell with different % of delamination . $R_x/a=5$, $R_y/a=5$, cross-ply-$(0/90)_n$

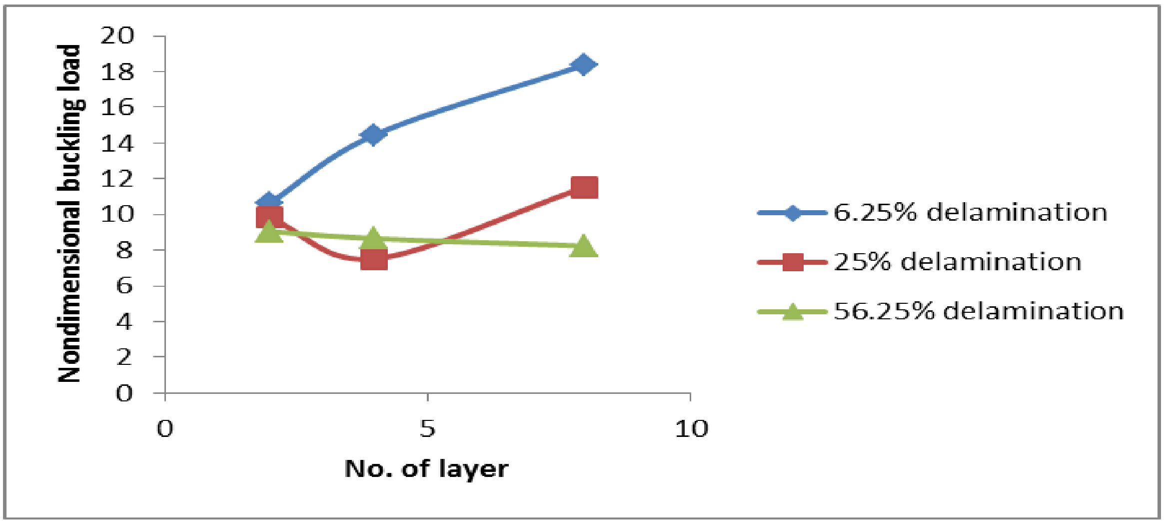

Figure 17: Variation of non-dimensional buckling load vs. no. of layers for simply supported composite plate with different % of delamination .Cross-ply-$(0/90)_n$

Numerical result for dynamic stability

Dynamic instability region are plotted for rectangular plates of stacking sequence $[(0/90)_2]_s$ for different delamination length. The laminates are made out of eight identical plied with material properties:

a=127mm, b=12.7mm, t=1.016mm, stacking sequence=(0/90/0/90/90/0/90/0).

In this study the boundary conditions is one of the short edges fixed and opposite edge loaded with dynamic buckling force. The non-dimensional excitation frequency $\Omega = \bar{\Omega} a^2 \sqrt{\rho / h^2 E_2}$ used throughout the dynamic instability studies. Where $\bar{\Omega}$ is the excitation frequency in radian /second.

CONCLUSION

A first order shear deformation theory based on finite element model has been developed for studying the instability region of mid plane delaminated composite plate and shell.
The following observations are made from this study:-

Vibration Study

- The effects of dynamic behaviour on delaminated composite plates and shells under free vibration conclude that for particular % of delamination, the natural frequencies increase with increase of number of layers due to effect

of bending-stretching coupling.
- With increase of aspect ratio,the natural frequency decreases.
- With increase of % delamination, the natural frequency decreases and it also observed the frequency of vibration increase with decrease of b/h ratio of cross ply panels with delamination. This is due to reduction in stiffness caused by delamination.
- With increase of % delamination the natural frequency decreases with different degrees of orthotropic.

Buckling study
- If no. of layers increases the non-dimensional buckling load increases and it also investigated that as % of delamination increases the non-dimensional buckling load decreases.
- If b/h ratio increases the non-dimensional buckling load increases for each delamination case and it also investigated that as % of delamination increases then on- dimensional buckling load decreases.
- The natural frequencies and then on-dimensional buckling load decrease with increase in delamination length. This is due to the reduction in stiffness caused by delamination.

Dynamic stability study

- The onset of instability occurs earlier with increase in percentage of de lamination.
- It also observed that with increases of number of layers the excitation frequency increases. This is due to increase of stiffness caused by bending-stretching coupling with increase of layers.
- The dynamic instability region occurs earlier with decrease of degree of orthotropy and due to decrease of de lamination the onset instability region shifted to lower frequency to higher frequency and also the width of instability regions increased with decrease of de lamination.
- The dynamic instability occurs much later with increase of aspect ratio and width of instability region increase with increase of aspect ratio for delaminated cross ply panel.
- With increase of static load factor the instability region tends to shift to lower frequencies and become wider showing destabilizing effect on the dynamic stability behavior of delaminated composite plate.

Further scope of study:
In the Present study, natural frequency, buckling load and dynamic stability of delaminated cross-ply composite plate and shell was determined numerically. The effect of various parameters like percentage of delamination area, number of layers, aspect ratio, degree of orthotropyand different side to thickness ratio was studied. The future scope of the present study can be extending as follows:

- Dynamicstabilityof multiple delaminated plateand shell can bestudied.
- The present studyis based on linear range of analysis. It can also extend for nonlinear analysis.
- Dynamicstabilityofcompositeplateswithcircular,elliptical,triangularshapeddelamination can be studied.
- The effect of damping on instability regions of delaminated composite plates and shells can be studied.

REFERENCES

[1]. **Abdullah[a,]*,Ferrero[b],Barrau[b],and Mouillet[c]**,(2007),"Development of a new finite element for composite delamination analysis" *Composites Science and Technology* 67, 2208–221.
[2]. **Alnefaie,** (2009), "Finite element modeling of composite plates with internal delamination" *Composite Structures* 90, 21–27.
[3]. **Bert, C.W. and Birman, V.** (1988) "Parametric instability of thick orthotropic circular cylindrical shells". *Acta Mechanica*, 71,61-76,.
[4]. **Chang, Hu, and Jane**, (1998), The vibration analysis of a delaminated composite plate underaxial load**,** *Mech. Struct. & Mach*, 26(2), 195-218.
[5]. **Chen *, Hong, and Liu**, (2004), the dynamic behavior of delaminated plates considering progressive failure process. *Composite Structures* 66, 459–466
[6]. **Chirica, Elena-Felicia and Beznea**, (2010), "Buckling analysis of the composite plates with delaminations" *Computational Materials Science* 50,1587–1591.
[7]. **Geubelle* and Baylor**, (1998), "Impact-induced delamination of composites using a 2D simulation" *Composites Part B* 29B, 589–602.
[8]. **Houa,*,andJeronimidisb**,(1999),"Vibration of delaminated thincomposite plates" *Composites:Part A* 30 ,989–995.
[9]. **Hu, Fukunaga, Kameyama, Aramaki, and Chang,** (2002), Vibration analysis of delaminated composite beams

and plates using a higher-order finite element,*International Journal of Mechanical Sciences* 44 1479–1503

[10]. **Hwang* and Mao,(**2001) "Failure of Delaminated Carbon/Epoxy Composite Plates under Compression" *Journal of Composite Materials* 35; 1634.

[11]. **Ju-fen, Gang, Howson and Williams**, (2003), "Reference surface element modelling of composite plate/shell delamination buckling and postbuckling" *Composite Structures 61* 255–264

[12]. **Ju, Lee and Lee,** (1995), "Finite element analysis of free vibration of delaminated composite plates" *Composites Engineering*, Vol. 5, No. 2, pp. 195-209

[13]. **Ju, .Lee and .Lee** (1995) "Free vibration of composite plates with delamination around cut-outs" *Composite Structure*, 31,177-183

[14]. **KüÇük,** (2004), an investigation on buckling behaviour of simplysupported woven steel, Reinforced thermoplastic laminated plates with lateral strip delamination. *Journal of Reinforced Plastics and Composites* 23; 209

[15]. **Lee and Chung**,(2010), " Finite element delamination model for vibrating composite spherical shell panels with central cut-outs" *Finite Elements in Analysis and Design* 46 247–256.

[16]. **Librescu, Khdeir and Frederick.** ,(1989). Shear deformation theory of laminated composite of shallow shell type panels and their response analysis. I: Free vibration and buckling. *Acta Mechnica*, 76, 1-33