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A Deterministic Inventory Model for Deteriorating Items with Biquadratic Demand, Linear Deterioration Rate, and Constant Holding Cost

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ABSTRACT

This paper uses a deterministic inventory model to study the deteriorating items using biquadratic demand, linear deterioration rate, and constant holding cost. Shortages are not allowed. An analytical solution is derived to minimize the total cost. Numerical examples are provided to validate the applicability of the model. Furthermore, sensitivity analysis examines the effect of changes in various parameters.

INTRODUCTION

Enterprises handling or dealing with deteriorating items need an effective inventory management system for its ceaseless and smooth functioning. Deterioration refers to the loss of product usability due to spoilage, decay, obsolescence, or damage over time. Effectively managing such inventory requires mathematical models that account for deterioration effects while optimizing costs and ensuring demand satisfaction.

Traditional inventory frameworks usually assume simple demand functions such as constant or exponentially varying demand. However, real-world demand patterns can be more complex, particularly in industries where demand fluctuates significantly over time. A biquadratic demand function, which captures nonlinear variations, provides a more realistic representation in scenarios such as seasonal product sales, perishable goods, and high-tech industries.

Classical inventory models frequently assume a constant deterioration rate. However, in practical scenarios, the deterioration rate varies due to environmental factors, storage conditions, or product-specific characteristics. A linear deterioration rate offers a more flexible and realistic approach compared to the constant deterioration assumption, making it more suitable for inventory systems dealing with time-sensitive products.

A deterministic inventory framework for the study of deteriorating items with a biquadratic demand function, a linear deterioration rate, and a constant holding cost has been developed for the current study. The purpose of the chosen study is to analyse the behaviour of the system under these conditions and derive optimal inventory policies that minimize costs while maintaining service efficiency. The proposed model is particularly relevant for industries dealing with perishable goods, pharmaceuticals, and consumer electronics, where both demand and deterioration vary over time.

The introductory part of the paper explains the model and emphasises the need of one such model in business enterprises. The objectives of the study are stated along with a literature review in the second section pertaining to the topic of the current study. Section 3 presents notations and assumptions for mathematical formulation. Section 4 formulates a mathematical model, incorporating biquadratic demand and, linear deterioration. Section 5 discusses the solution methodology and optimal policies. The final section presents the sensitivity analysis of the proposed model with numerical illustrations. The findings of the study and the scope for future research are also presented.

The current study adds to the emerging field of inventory modelling by integrating a biquadratic demand function and a linear deterioration rate, offering more realistic inventory management. The findings provide valuable insights for businesses and managers seeking to optimise inventory decisions for deteriorating products.

LITERATURE REVIEW

Managing inventory for deteriorating items is a critical challenge in industries like food, pharmaceuticals, and perishables. Researchers have developed various models to optimize stock levels while considering factors like demand patterns, holding costs, and deterioration rates.

The study of deteriorating items with inventory model began with Whitin (1957), who laid the foundation for inventory management. Later, Dave and Patel (1981) introduced a model where demand changes over time, making it more realistic. Harriga (1996) further improved the EOQ model by considering time-varying demand, which is useful for industries like pharmaceuticals and perishables.

Mishra (2012) developed a model with ramp-type demand and linear deterioration, capturing situations where demand increases or decreases gradually. Bakker et al. (2011) provided a broader review of inventory systems with deterioration, summarizing different approaches and their applications.

Mishra and Singh (2012) introduced a model that includes time-dependent demand and changing holding casts, recognizing that storage costs vary over time. Similarly, Kumar and Ravendra (2015) focused on perishable items, incorporating shortages and time-dependent demand.

More recent studies have explored complex demand and deterioration patterns. Sharma (2021) developed a model with biquadratic demand and time-dependent deterioration, while Soni and Kumar (2021) used a polynomial function to model demand more accurately. Shelly and Kumar (2021) considered how holding costs change with demand, making the model more practical for real-world applications.

Some researchers have used statistical methods to improve deterioration modelling. Suman and Kumar (2022) applied the Weibull distribution to represent varying deterioration rates, which is useful for items like food and medicine. Jitendra (2022) incorporated price-dependent demand, showing how pricing strategies influence inventory decisions.

An EOQ model with quadratic time-dependent deterioration and shortages was developed in the recent years by Pappu Kumar and Tripathi (2024) helping businesses manage perishable stock effectively. Ramesh and Ganesh Kumar (2024) analyzed inventory models for dual-warehouse systems, considering different deterioration rates across locations.

Overall, research in this field has progressed from basic models to advanced systems that account for changing demand, shortages, and complex deterioration patterns. These studies help industries like healthcare, food, and perishables manage their inventory efficiently.

Notations and Assumptions
The following notations and assumptions are used for mathematical formulation:

Notations
$I(t)$: Inventory level at any time t, $0 \leq t \leq T$.
O_c: Ordering cost per order.
$D(t)$: $a + bt + ct^2 + dt^3 + et^4$, Biquadratic demand rate, where a,b,c,d,e are constants.
$\theta(t) = \theta_0 + \theta_1 t$, Linear deterioration rate.
h_c: Holding cost
T : Ordering cycle length.
I_0 : Maximum inventory level at t=0.
TC(T): Total cost per unit time.
Q : The Economic order quantity.
C :Cost price per unit.

Assumptions

- The rate of demand is deterministic and is a biquadratic function of time.
- Holding cost is considered to be constant.
- Deterioration is a linear function of time.
- Shortages are not allowed.
- Lead time is zero.
- Rate of replenishment is infinite
- The time horizon is infinite.
- The model is developed for single-item inventory only.
- There is no repair of the deteriorated items or replenishment during the given cycle.

Mathematical formulation and solution of the model

Let $I(t)$ denote the inventory level at any time t. The decrease in inventory level results from both demand and deterioration. Over the period $[0, T]$, the inventory level steadily decreases and eventually reaches zero at $t = T$.

The following differential equation represents the inventory level over the interval [0,T]

$$\frac{dI(t)}{dt} + (\theta_0 + \theta_1 t)I(t) = -(a + bt + ct^2 + dt^3 + et^4) ------(1)$$

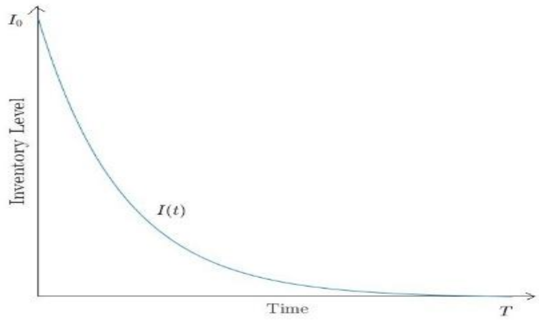

Figure 1: Graphical Representation of Inventory System

The solution is

$$\begin{aligned} I(t) = {} & a(T-t) + \frac{b}{2}(T^2 - t^2) + \frac{c}{3}(T^3 - t^3) + \frac{d}{4}(T^4 - t^4) + \frac{e}{5}(T^5 - t^5) \\ & + \theta_0 \left\{\frac{a}{2}(T^2 - t^2) + \frac{b}{3}(T^3 - t^3) + \frac{c}{4}(T^4 - t^4) + \frac{d}{5}(T^5 - t^5) + \frac{e}{6}(T^6 - t^6)\right\} \\ & + \theta_1 \left\{\frac{a}{6}(T^3 - t^3) + \frac{b}{8}(T^4 - t^4) + \frac{c}{10}(T^5 - t^5) + \frac{d}{12}(T^6 - t^6) + \frac{e}{14}(T^7 - t^7)\right\} \\ & - \theta_0 \left\{a(Tt - t^2) + \frac{b}{2}(T^2 t - t^3) + \frac{c}{3}(T^3 t - t^4) + \frac{d}{4}(T^4 t - t^5) + \frac{e}{5}(T^5 t - t^6)\right\} \\ & - \frac{\theta_1}{2}\left\{a(Tt^2 - t^3) + \frac{b}{2}(T^2 t^2 - t^4) + \frac{c}{3}(T^3 t^2 - t^5) + \frac{d}{4}(T^4 t^2 - t^6) + \frac{e}{5}(T^5 t^2 - t^7)\right\} \\ & ------(2) \end{aligned}$$

Maximum inventory level is obtained by putting $t = 0$ in (2)

$$\begin{aligned} I_0 = I(0) = {} & aT + \frac{bT^2}{2} + \frac{cT^3}{3} + \frac{dT^4}{4} + \frac{eT^5}{5} + \theta_0 \left(\frac{aT^2}{2} + \frac{bT^3}{3} + \frac{cT^4}{4} + \frac{dT^5}{5} + \frac{eT^6}{6}\right) \\ & + \theta_1 \left(\frac{aT^3}{6} + \frac{bT^4}{8} + \frac{cT^5}{10} + \frac{dT^6}{12} + \frac{eT^7}{14}\right) \end{aligned}$$

Ordering cost

Ordering cost, $OC = o_C$

Holding cost

The total holding cost for the period $[O, T]$ is $HC = h_c \int_O^T I(t)dt$

$$HC = h_c\left[\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}+\frac{dT^5}{5}+\frac{eT^6}{6}+\theta_0\left(\frac{aT^3}{6}+\frac{bT^4}{8}+\frac{cT^5}{10}+\frac{dT^6}{12}+\frac{eT^7}{14}\right)\right.$$
$$\left.+\theta_1\left(\frac{aT^4}{12}-\frac{bT^5}{15}-\frac{cT^6}{18}-\frac{dT^7}{21}-\frac{eT^8}{24}\right)\right]$$

Total demand over the cycle period $[O,T] = \int_O^T D(t)dt$

$$= aT+\frac{bT^2}{2}+\frac{cT^3}{3}+\frac{dT^4}{4}+\frac{eT^5}{5}$$

No of deteriorated units $=$ Initial order quantity $-$ Total demand in the cycle period $[O,T]$

$$= I_0 - \int_O^T D(t)dt$$
$$= \theta_0\left(\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}+\frac{dT^5}{5}+\frac{eT^6}{6}\right)+\theta_1\left(\frac{aT^3}{6}+\frac{bT^4}{8}+\frac{cT^5}{10}+\frac{dT^6}{12}+\frac{eT^7}{14}\right)$$

Deterioration cost:

The cost of deterioration cost per cycle is $DC = c \times No\ of\ deteriorated\ units$

$$= c\left\{\theta_0\left(\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}+\frac{dT^5}{5}+\frac{eT^6}{6}\right)+\theta_1\left(\frac{aT^3}{6}+\frac{bT^4}{8}+\frac{cT^5}{10}+\frac{dT^6}{12}+\frac{eT^7}{14}\right)\right\}$$

Total cost:

$$\text{Total variable cost} = Ordering\ cost(OC) + \text{Holding cost }(HC) + \text{Deterioration cost }(DC)$$

The total variable cost per unit time $is\ TC(T) = \frac{1}{T}[OC+HC+DC]$

$$TC(T) = \frac{o_c}{T} + h_c\left\{\frac{aT}{2}+\frac{bT^2}{3}+\frac{cT^3}{4}+\frac{dT^4}{5}+\frac{eT^5}{6}+\theta_0\left(\frac{aT^2}{6}+\frac{bT^3}{18}+\frac{cT^4}{10}+\frac{dT^5}{12}+\frac{eT^6}{14}\right)\right.$$
$$\left.+\theta_1\left(\frac{aT^3}{12}+\frac{bT^4}{15}+\frac{cT^5}{18}+\frac{dT^6}{21}+\frac{eT^7}{24}\right)\right\}$$
$$+ c\left\{\theta_0\left(\frac{aT}{2}+\frac{bT^2}{3}+\frac{cT^3}{4}+\frac{dT^4}{5}+\frac{eT^5}{6}\right)+\theta_1\left(\frac{aT^2}{6}+\frac{bT^3}{8}+\frac{cT^4}{10}+\frac{dT^5}{12}+\frac{eT^6}{14}\right)\right\}$$

$$\frac{dTC(T)}{dT} = -\frac{o_c}{T^2} + h_c\left\{\frac{a}{2}+\frac{2bT}{3}+\frac{3cT^2}{4}+\frac{4dT^3}{5}+\frac{5eT^4}{6}+\theta_0\left(\frac{aT}{3}+\frac{3bT^2}{8}+\frac{2cT^3}{5}+\frac{5dT^4}{12}+\frac{3eT^5}{7}\right) + \theta_1\left(\frac{aT^2}{4}+\frac{4bT^3}{15}+\frac{5cT^4}{18}+\frac{2dT^5}{7}+\frac{7eT^6}{24}\right)\right\} + c\left\{\theta_0\left(\frac{a}{2}+\frac{2bT}{3}+\frac{3cT^2}{4}+\frac{4dT^3}{5}+\frac{5eT^4}{6}\right) + \theta_1\left(\frac{aT}{3}+\frac{3bT^2}{8}+\frac{2cT^3}{5}+\frac{5dT^4}{12}+\frac{3eT^5}{7}\right)\right\} ------(3)$$

Equating equation (3) to zero and simplifying by multiplying both sides by $2520T^2$ in order to determine T that minimizes the total cost per unit time as follows:

$$-2520o_c + h_c\{1260bT^2 + 1680T^3 + 1890T^4 + 2016T^5 + 2100T^6 + \theta_0(840T^3 + 945T^4 + 1008T^5 + 1050T^6 + 1080T^7) + \theta_1(630aT^4 + 672bT^5 + 700cT^6 + 720dT^7 + 735eT^8)\} + c\{\theta_0(1260aT^2 + 1680bT^3 + 1890cT^4 + 2016dT^5 + 2100eT^6) + \theta_1(840aT^3 + 945bT^4 + 1008cT^5 + 1050dT^6 + 1080eT^7)\}\} = 0$$

$$\frac{d^2TC(T)}{dT^2} = \frac{2o_c}{T^3} + h_c\left\{\frac{2b}{3}+\frac{3cT}{2}+\frac{12dT^2}{5}+\frac{10eT^3}{3}+\theta_0\left(\frac{a}{3}+\frac{3bT}{4}+\frac{6cT^2}{5}+\frac{5dT^3}{3}+\frac{15eT^4}{7}\right) + \theta_1\left(\frac{aT}{2}+\frac{4bT^2}{5}+\frac{10cT^3}{9}+\frac{10dT^4}{7}+\frac{7eT^5}{4}\right)\right\} + c\left\{\theta_0\left(\frac{2b}{3}+\frac{3cT}{2}+\frac{12dT^2}{5}+\frac{10eT^3}{3}\right)+\theta_1\left(\frac{a}{3}+\frac{3bT}{4}+\frac{6cT^2}{5}+\frac{5dT^3}{3}+\frac{15eT^4}{7}\right)\right\}$$

Solution Methodology

To minimize the average cost per unit of time, the following equation must be solved to obtain T.

$$\frac{d(TC)}{dT} = 0 \qquad ----- \text{(4)}$$

with the optimality condition

$$\frac{d^2(TC)}{dT^2} > 0 ------(5)$$

Algorithm:

To obtain the optimal results for the total cost (TC), initial stock (Q) and cycle length (T), an iterative algorithm is suggested.
Step 1: Enter the value of every parameter needed for the model.

Step 2: Evaluate the cycle length T using equation (4) and assume this value as T^*.

Step 3: Check the optimality condition using (5).

Step 4: If the condition in step 3 is met, proceed to step 5; if not, go back to steps 1 through 3 for various parameter values.

Step 5: Find the optimal initial inventory stock I_0 or Q and TC values for T^*.

Step 6: Stop once the optimal values are found.

Numerical example
A numerical example to demonstrate the model's practical implementation is given below:

The solution is obtained by applying the algorithm given above. Here MATLAB is used to perform the computations.

Example: For the given model, the values of various parameters are taken as follows:

a = 87 , b =45, c=36, d=45, e =19, $\theta_0 = 0.1$, $\theta_1 = 0.2$, O_c =120, h_c =15.

The optimal values are obtained as follows

Optimal cycle length (T) =0.319910 (116.77 days)

Minimum total cost per unit time (TC) =Rs.685.57

Economic order quantity (Q) =31.28 units.

Sensitivity Analysis
Illustration of the proposed model with a sensitivity analysis is presented here.

To present the difference in values of various parameters on optimal solution, the sensitivity analysis is carried out and the results are presented in the tables given below. The results show the impact of values on various parameters on the specified cycle length (T), total cost (TC), and economic order quantity (Q).

Table 1: Effect of "a" on T, TC, and Q

Change in a	T	TC	Q
-20%	0.3417	629.1745	27.6595
-10%	0.3304	657.8647	29.5141
10%	0.3102	712.3792	32.9543
20%	0.3013	738.3543	34.5567

Table 2: Effect of "b" on T, TC, and Q

Change in p meter b	T	TC	Q
-20%	0.3244	679.4684	31.2844

-10%	0.3221	682.5425	31.2810
10%	0.3178	688.5618	31.2712
20%	0.3157	691.5100	31.2683

Table 3: Effect of "c" on T, TC and Q

Change in c	T	TC	Q
-20%	0.3280	670.2973	32.0890
-10%	0.3238	677.9916	31.6712
10%	0.3161	693.0470	30.8921
20%	0.3125	700.4179	30.5240

Table 4: Effect of "d" on T, TC and Q

Change in d	T	TC	Q
-20%	0.3204	685.1980	31.3071
-10%	0.3201	685.3859	31.2894
10%	0.3196	685.7597	31.2580
20%	0.3194	685.9457	31.2407

Table 5: Effect of "e" on T, TC and Q

Change in e	T	TC	Q
-20%	0.3200	685.5309	31.2812
-10%	0.3199	685.5520	31.2783
10%	0.3199	685.5942	31.2726
20%	0.3198	685.6152	31.2698

Table 6: Effect of "θ_0" on T, TC and Q

Change in θ_0	T	TC	Q
-20%	0.3250	673.5633	31.7401
-10%	0.3224	679.5966	31.5048
10%	0.3174	691.4942	31.0519
20%	0.3150	697.3616	30.8338

Table 7: Effect of "OC" on T, TC and Q

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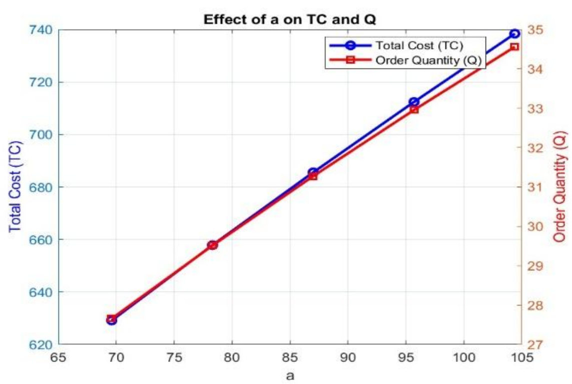
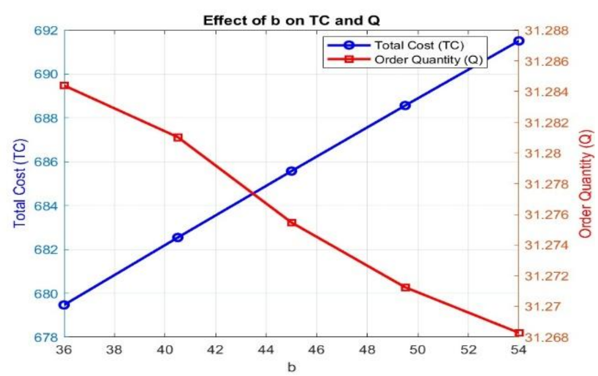
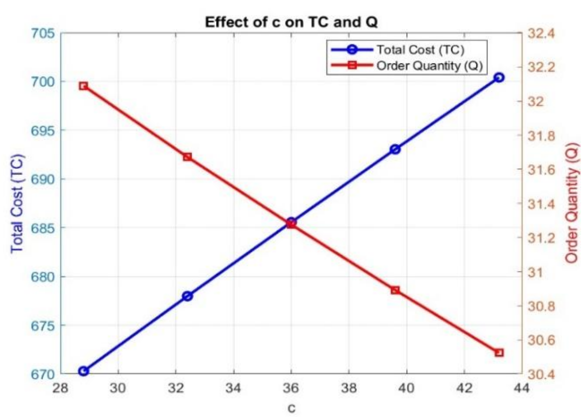
Change in a	T	TC	Q
-20%	0.2916	607.1080	28.1630
-10%	0.3063	647.2481	29.7685
10%	0.3326	722.3508	32.6999
20%	0.3446	757.7888	34.0530

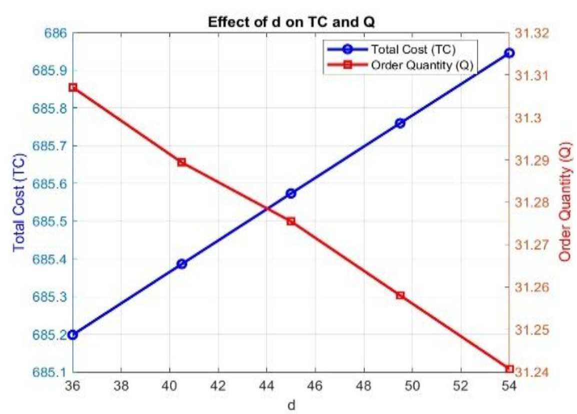
Table 8: Effect of "HC" on T, TC and Q

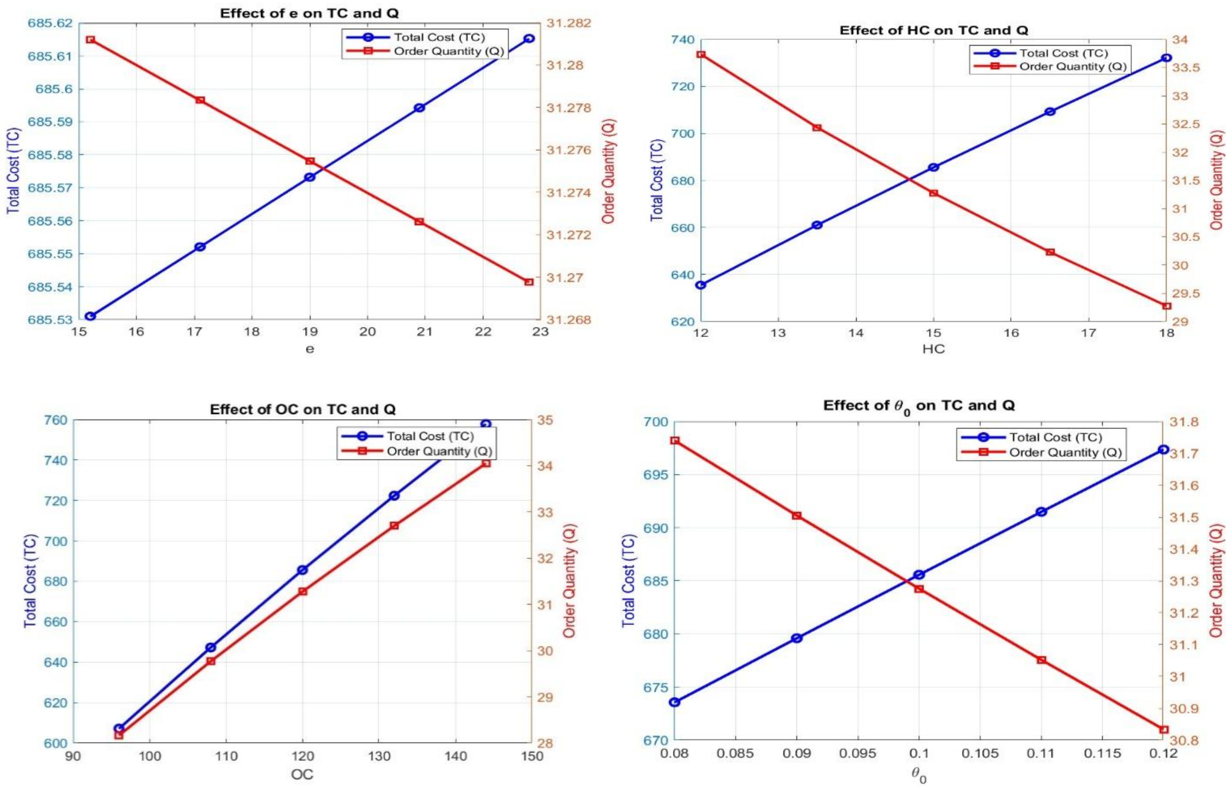
Change in a	T	TC	Q
-20%	0.3418	635.4878	33.7365
-10%	0.3303	661.0193	32.4379
10%	0.3105	709.2520	30.2285
20%	0.3018	732.1419	29.2767











Figures: 1 to 9

Analysis and Interpretation

1. As ‘a’ increases, Q and TC increases whereas ‘T’ decreases, this suggests that higher first order demand leads to a shorter cycle time and higher order quantity, increasing total costs.
2. Increase in ‘b’ slightly decreases T and Q whereas slight increase in the value of Q,this minor impact shows that second order demand variation does not significantly alter inventory behaviour.
3. As ‘c’ increases, slight decrease in T and Q whereas TC increases
4. As ‘d’ increases, slight decrease in T & Q and very slight increase in TC.
5. As ‘e’ increases, almost no change in T, slight increase in TC and slight decrease in Q.
6. As "Q_0" increases, cycle length and quantity ordered decreases whereas total cost increases.
7. As "Q_1" increases, T & Q decreases, whereas TC increases.
8. As "O_c" increases, T, TC & Q increases significantly.
9. As "h_c" increases, small decrement in T and Q but significant increase in TC.

CONCLUSION

The model developed for this research incorporating a biquadratic demand function, a linear deterioration rate, and a constant holding cost highlights impact of demand and deterioration on optimal ordering policies, providing valuable insights for businesses managing perishable goods.

The results emphasize the importance of accurately modelling these factors to minimize costs and improve efficiency. Sensitivity analysis was conducted to examine how key parameters - such as demand coefficients, deterioration rate, and holding cost affect the total cost and inventory decisions. The findings indicate that small variations in these parameters can significantly influence optimal policies.

Future research could explore the inclusion of dynamic demand patterns, time varying deterioration rates, variable holding costs partial and complete backlogging and multi product systems.

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